Chapter 2

Logic and Reasoning

2.1 Conditional Statements

✓ Example: Use *p:* **you see stars**, and *q:* **it is night** to write a conditional statement and a related conditional statements. Make sure to write the symbols that correspond to each statement.

Conditional: ____

____

Negation of *p*: ____

Converse:

____

____

Inverse: ____

____

Contrapositive: ____

____

✓ Example: Use *p:* **x is an even number**, and *q:* **x is divisible by 2.** to write a conditional statement and a related conditional statements. Make sure to write the symbols that correspond to each statement.

Conditional: ____

____

Negation of *p*: ____

Converse: ____

____

Inverse: ____

____

Contrapositive: ____

____

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Use the conditional statement to write a biconditional statement.

1. If x = -2, then 3x + 8 = 2

2. If an angle is straight, then it measures 180°.

Write each of the following biconditional statement as a conditional statement and its converse.

3. It is a bird if and only if it has a beak.

4. It's the weekend if and only if it is Saturday.

Rewrite the statements as a single biconditional statement.

5. If today is the 4th Thursday of November, then it is Thanksgiving in the United States.
If today is Thanksgiving in the United States, then it is the 4th Thursday in November.

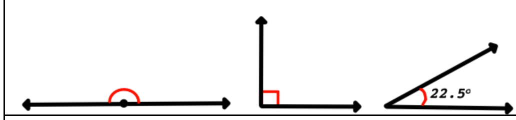
6. If points are collinear, then they all lie in one line. If points all lie in one line, then they are collinear.

2.2 Inductive and Deductive Reasoning

Conjecture:

Inductive Reasoning	Deductive Reasoning

Example: Use inductive reasoning to determine the pattern and make a conjecture. Then, state the next two numbers, figures, or letters.

Pattern	Conjecture	Next Two Items
1. 		
2. -7, -2, 3, 8, ...		
3. A, D, H, L, ...		

Example: Make and test a conjecture:

4. The product of a negative integer and a positive integer	5. The difference of any two odd integers

Counterexample:

Example: Find a counterexample to shoe that the conjecture is false.

6. The value of x^2 is always greater than the value of x.	7. The sum of two numbers is always greater than their difference.

Law of Detachment	Law of Syllogism
A ________________ of reasoning. **If a _________________ is true and its _______________ is true, then its _______________ is _______________.** • Statement 1: If p, then q. • Statement 2: p ○ Conclusion: q	Used when you have ______ ______________ and the ______________ of one matches the ________________ of the other. • Statement 1: If p, then q. • Statement 2: If q, then r. • Then... Statement 1: If p, then r.

State the law of logic that is being illustrated. Write (D) for law of detachment. or (S) for law of Syllogism.

______ 5. If Amy receives a 83% or higher her Geometry final exam, then she will pass the class. Amy received 42/50 on her Geometry final exam. Therefore, Amy passed the class.

______ 6. If Cedric plays in a big game, then he gets nervous. If Cedric gets nervous, then he performs well. Therefore, if Cedric plays in a big game, then he performs well.

______ 7. If a triangle has two angles that measure 60° , then the triangle is equiangular. If a triangle is equiangular, then it is also equilateral. Therefore, if two angles in ΔABC are both 60°, then ΔABC is also equilateral.

______ 8. If $x > 9$, then $-4x + 2 > -34$. The value of x is 12 so, $-4x + 2 > -34$.

Decide if inductive reasoning (IR) or deductive reasoning (DR) is used to reach a conclusion. Explain.

______ 9. For the past three Wednesdays, the cafeteria has served macaroni and cheese for lunch. Dana concludes that the cafeteria will serve macaroni and cheese for lunch this Wednesday.

______ 10. If you live in Nevada and are between the ages of 16 and 18, then you must take driver's education to get your license. Marcus lives in Nevada, is 16 years old, and has his driver's license. Marcus took driver's education.

Determine if it is possible to use the law of Syllogism to write a new conditional statement. Explain.

______ 10. If Lavonne gets money, she gives half of it to Sid. If Sid gets money, he gives half of it to Lavonne.

Examples:

Use the Law of Detachment to determine what you can conclude from the given information, if possible.

1. If a figure is rhombus, then it is a quadrilateral. You know that LMNP is a rhombus.

2. If angles have the same measure, then they are congruent. I know that $m\angle A = m\angle B$.

Examples:

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

3. If it is raining today, then soccer practice is cancelled. If soccer practice is cancelled, then you can go to the mall after school.

4. If Tim gets stung by a bee, then he will get very ill. If he gets very ill, then he will go to the hospital.

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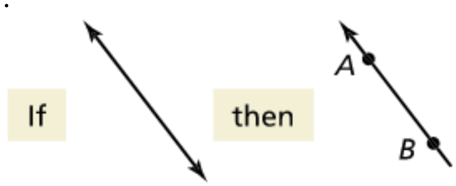
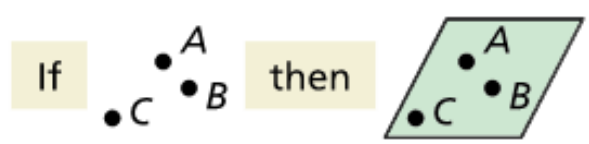
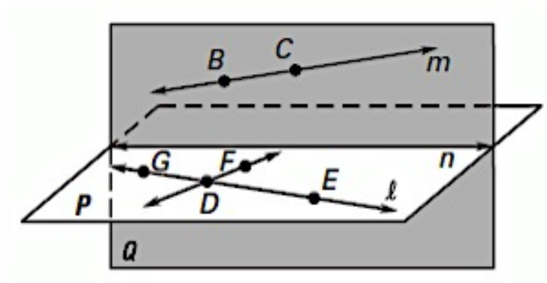
2.3 Postulates and Diagrams

Seven Postulates involving points, lines, and planes.							
Name IT	Two Point Postulate	Line-Point Postulate	Line Intersection Postulate	Three Point Postulate	Plane-Point Postulate	Plane-Line Postulate	Plane Intersection Postulate
Draw IT							
Put IT in words	Through any ___ points, there exists _______ _____ line.	A ________ Contains at least _______ _________.	If two lines _________, then their intersection is _______ ____point.	Through any ________ noncollinear ________, there exists exactly ___ ________.	A _______ contains at least _______ noncollinear ________.	If _______ ________ lie in a ________, then the line containing them lies in the plane.	If ________ __________ intersect, then their ________ is a _______.

State the postulate illustrated by the diagram.

1.	2.
	

Use the diagram to write examples of each postulates.



3. Plane-Point Postulate	4. Two Point Postulate
5. Plane Intersection Postulate	6. Plane-Line Postulate

Choose all the statements about the diagram on the right that you can assume to be true.

A. $AH \cong HB$

B. $\overline{ED} \perp \overline{AB}$

C. $EH \cong FB$

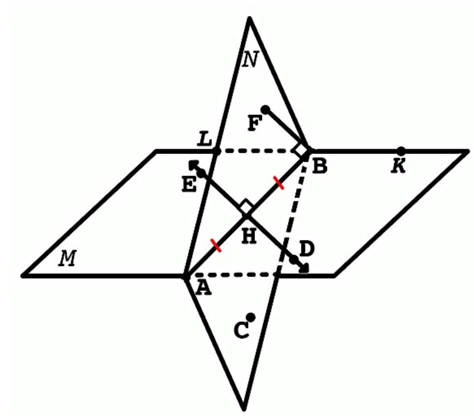
D. Points B, F, and C are coplanar.

E. Plane M intersects Plane N at $\overleftrightarrow{AB}$

F. Points F, H, and C are Collinear

G. $\angle FBL$ and $\angle KBD$ are vertical Angles.

H. $\angle ALK$ is a right angle.



Perpendicular (⊥):

Line perpendicular to a plane:

Sketch a diagram of the description.

7. $\overrightarrow{VX}$ intersecting $\overrightarrow{UW}$ at V so that $\overrightarrow{VX}$ is perpendicular to $\overrightarrow{UW}$ and U, V, and W are collinear.	8. S is on line q and is the midpoint of $\overline{NP}$. Line q intersects $\overline{NP}$. Points R, S, T are collinear on Plane M.

2.4 Algebraic Reasoning

Distributive Property:

Match the statement with the Property of Equality

_______	**1.** If $JK = PQ$ and $PQ = ST$, then $JK = ST$.	**A.** Addition property
_______	**2.** If $m\angle S = 30°$, then $5° + m\angle S = 35°$.	**B.** Reflexive property
_______	**3.** If $AB + CD = EF + CD$, then $AB = EF$.	**C.** Substitution property
_______	**4.** $AB = AB$	**D.** Transitive property
_______	**5.** If $ST = 2$, then $ST + TU = 2 + TU$.	**E.** Symmetric property
_______	**6.** If $m\angle K = 45°$, then $3(m\angle K) = 135°$.	**F.** Multiplication property
_______	**7.** If $m\angle P = m\angle Q$, then $m\angle Q = m\angle P$.	**G.** Subtraction property

Solve each equation. Justify each step.

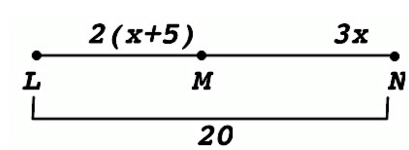
1. Given: $6x - 11 = 25$ Prove: x = 6	2. Given: $-2(p + 4) = 10p - 16$ Prove: $p = \frac{2}{3}$

Rewrite the formula for the variable. Justify your steps.

3. $A = \frac{1}{2}bh; b$

Then, find value of the base of the triangle when the area is 952 ft^2 and the height is 56 ft.

4. Given: LM = 2(x + 5)
MN = 3x
LN = 20
Prove: x = 2



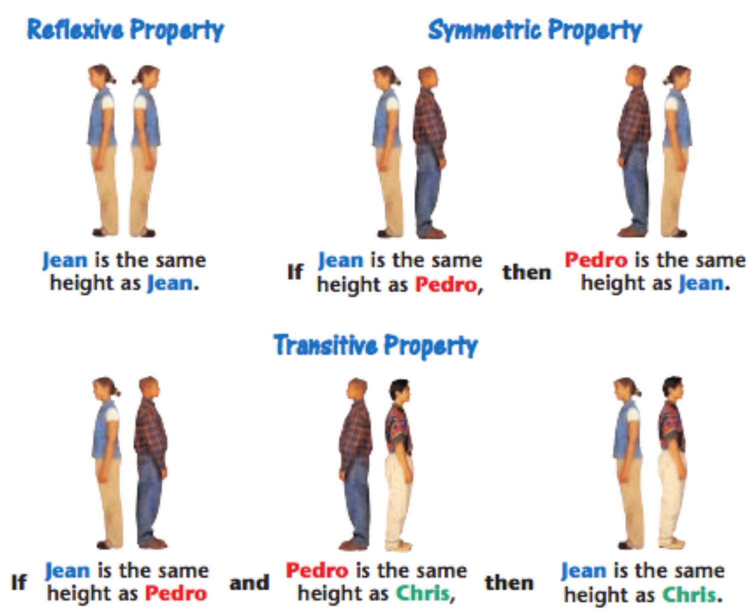
2.5 Proving Statements about Segments

Proof two-column

Proof

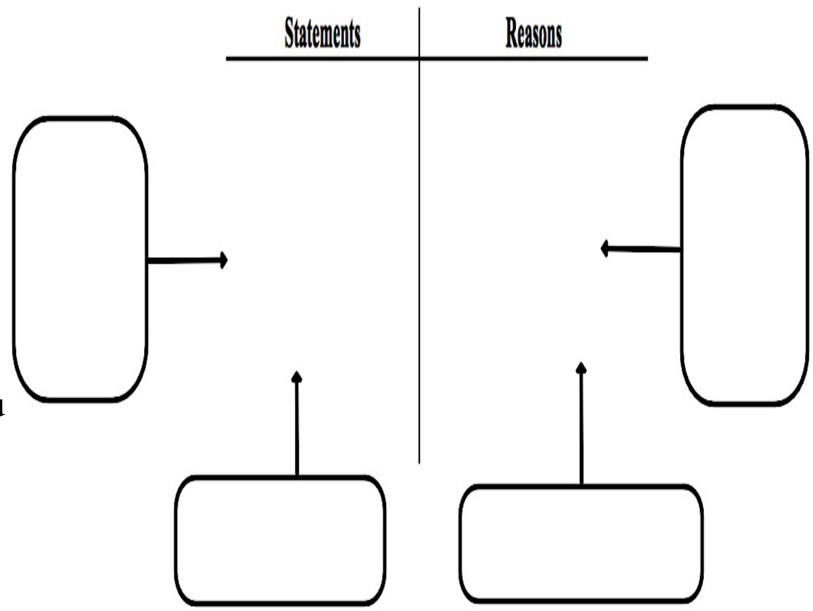
Theorem

PROPERTY	SEGMENT	ANGLE
REFLEXIVE		
SYMMETRIC		
TRANSITIVE		



Making a two-column proof:

In a proof, you make ___ statement at a time until you reach the _____________. Because you make statements based on _____________, you are using ____________ reasoning. Usually the __________ statement-and-reason pair you write is given information.



Examples:

A. Match the statement with the property that it illustrates

1. $m\angle DEF = m\angle DEF$

2. If $\overline{PQ} \cong \overline{ST}$, then $\overline{ST} \cong \overline{PQ}$.

3. $\overline{XY} \cong \overline{XY}$

4. If $\angle J \cong \angle K$ and $\angle K \cong \angle L$, then $\angle J \cong \angle L$.

5. If $PQ = QR$ and $QR = RS$, then $PQ = RS$.

6. If $m\angle X = m\angle Y$, then $m\angle Y = m\angle X$.

A. Symmetric Property of Equality

B. Reflexive Property of Equality

C. Transitive Property of Equality

D. Reflexive Property of Congruence

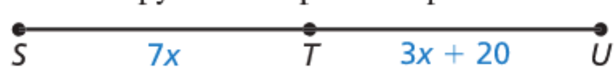
E. Symmetric Property of Congruence

F. Transitive Property of Congruence

B. Fill in the two-column proof.

Six steps of a two-column proof are shown. Copy and complete the proof.

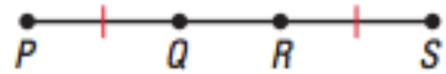
Given T is the midpoint of $\overline{SU}$.

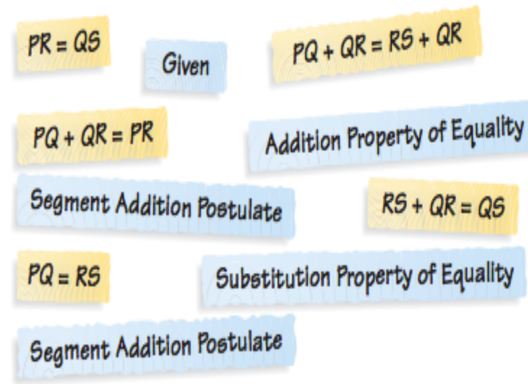


Prove $x = 5$

STATEMENTS	REASONS
1. T is the midpoint of $\overline{SU}$.	**1.** ______________________
2. $\overline{ST} \cong \overline{TU}$	**2.** Definition of midpoint
3. $ST = TU$	**3.** Definition of congruent segments
4. $7x = 3x + 20$	**4.** ______________________
5. ______________________	**5.** Subtraction Property of Equality
6. $x = 5$	**6.** ______________________

C. In the diagram, PQ = RS. Copy the diagram and arrange the statements and reasons in order to make a logical argument to show that PR = QS.





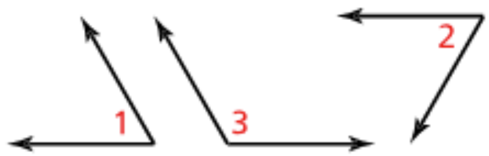
Statements	**Reasons**

D. Write a two-column proof.

Given: $\angle 1$ are supplementary in $\angle 3$

$\angle 2$ are supplementary in $\angle 3$

Proof: $\angle 2 \cong \angle 1$



Statements	**Reasons**
1.	
2.	
3.	
4.	
5.	
6.	
7.	

2.6 Proving Geometric Relationships

Flowchart proof, or flow proof

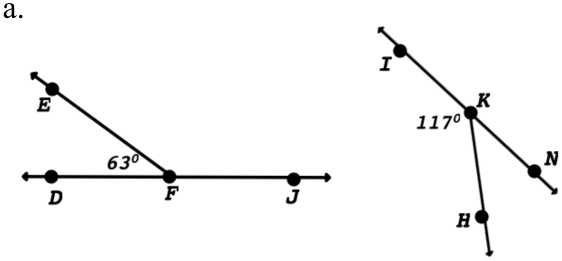
Paragraph proof

Congruent Complement Theorem	Congruent Supplement Theorem
Right Angles Congruence Theorem	**Linear Pair Postulate**
Vertical Angles Congruence Theorem	

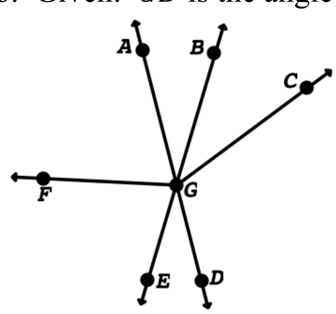
Examples:

1. Identify the pair(s) of congruent angles in the figures. Explain how you know they are congruent.

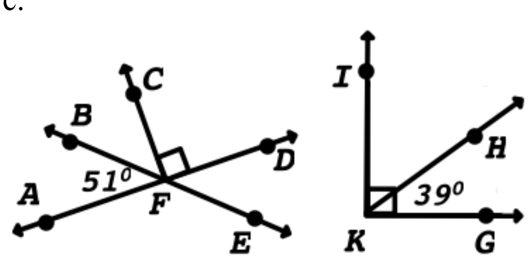
a.



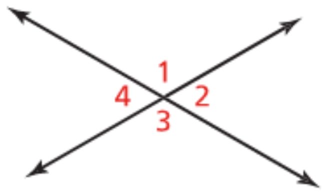
b. Given: $\overrightarrow{GB}$ is the angle bisector of $\angle AGC$.



c.

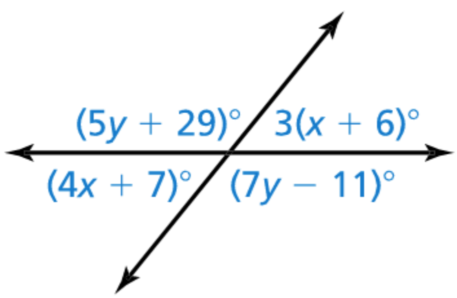


2. Use the diagram and the given information to find the other three angles.

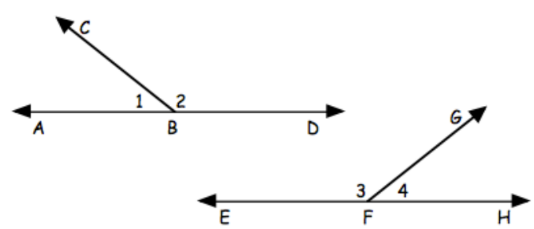


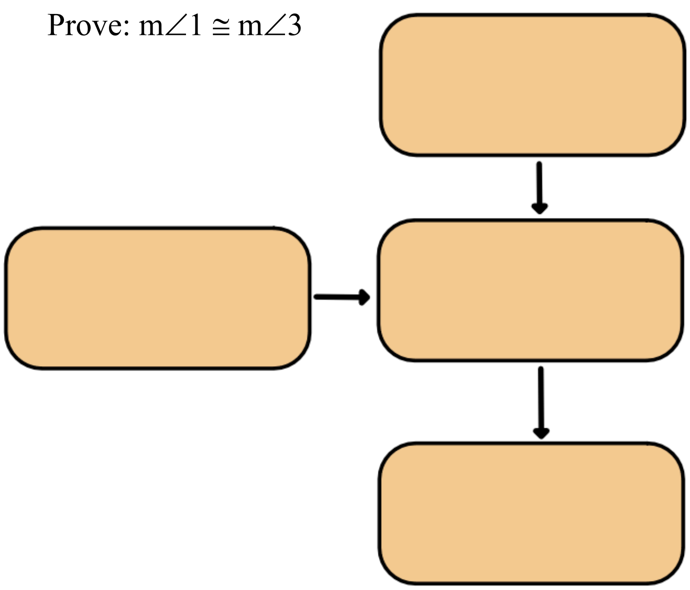
a. $m\angle 1 = 117°$	$m\angle 2 =$ ______° $m\angle 3 =$ ______° $m\angle 4 =$ ______°
b. $m\angle 2 = 59°$	$m\angle 1 =$ ______° $m\angle 3 =$ ______° $m\angle 4 =$ ______°

3. Solve for x and y.



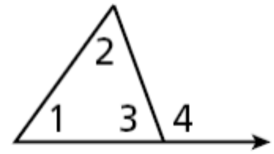
4. Complete the flowchart proof. Then write a two-column proof.
   Given: $\angle 2 \cong \angle 4$
   Prove: $m\angle 1 \cong m\angle 3$





Statements	Reasons
1. $\angle 2 \cong \angle 4$	1.
2.	2. Linear Pair Postulate
3. $\angle 1 \cong \angle 3$	3.
4.	4. Definition of $\cong$ angles

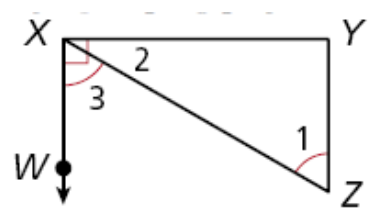
3. Use the given paragraph proof to complete a two-column proof.
   Given: m∠1 + m∠2 = m∠4
   Prove: m∠3 + m∠1 + m∠2 = 180°



Paragraph Proof: It is given that m∠1 + m∠2 = m∠4. ∠3 and ∠4 are supplementary by the Linear Pair Theorem. So m∠3 + m∠4 = 180° by definition. By Substitution, m∠3 + m∠1 + m∠2 = 180°.

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.

4. Use the given two-column proof to write a paragraph proof.
   Given: ∠WXY is a right angle.
   ∠1 ≅ ∠3
   Prove: ∠1 and ∠2 are complementary.



Statements	Reasons
1. ∠WXY is a right angle.	1.
2.	2. Definition of right angle
3.	3. Angle Addition Postulate
4. m∠2 + m∠3 = 90^0	4.
5.	5. Given
6. m∠1 = m∠3	6.
7.	7.
8. ∠1 and ∠2 are complementary	8.

Paragraph Proof: Since ∠WXY is a right angle, m∠WXY = 90° by the ________________________. By the Angle Addition Postulate, _____________________________. By _____________, m∠2 + m∠3 = 90°. Since ∠1 ≅ ∠3, m∠1 = m∠3 by the definition of ______________________. Using substitution, m∠2 + m∠1 = 90°. Thus, by the definition _____________________________, ∠1 and ∠2 are complementary.