

Section 7.2. Solving Linear Systems by Substitution

Solving a Linear System by Substitution

1. Solve one of the equations for one of its variables.
2. Substitute the expression from Step 1 into the other equation and solve for the other variable.
3. Substitute the value from Step 2 into the revised equation from Step 1 and solve.
4. Check the solution in each of the original equations.

EXAMPLES

1. $\begin{cases} 3x + y = 5 \\ x - y = 10 \end{cases}$ $\begin{cases} 3(3) - 4 = 5 \\ 9 - 4 = 5 \\ 5 = 5 \end{cases}$ $\begin{cases} 6 + 9 = 15 \\ 15 = 15 \\ 15 = 15 \end{cases}$ $\begin{cases} 3x + 2x - 10 = 5 \\ 5x - 10 = 5 \\ 5x = 15 \\ x = 3 \end{cases}$ $\begin{cases} 2(3) + 4y = 15 \\ 6 + 4y = 15 \\ 4y + 6 = 15 \\ 4y = 9 \\ y = \frac{9}{4} \end{cases}$ $\begin{cases} 3x + 2x - 10 = 5 \\ 5x - 10 = 5 \\ 5x = 15 \\ x = 3 \end{cases}$ $\begin{cases} 3(3) - 4 = 5 \\ 9 - 4 = 5 \\ 5 = 5 \end{cases}$ $\begin{cases} 6 + 9 = 15 \\ 15 = 15 \\ 15 = 15 \end{cases}$ $\begin{cases} 3x + 2x - 10 = 5 \\ 5x - 10 = 5 \\ 5x = 15 \\ x = 3 \end{cases}$ $\begin{cases} 2(3) + 4y = 15 \\ 6 + 4y = 15 \\ 4y + 6 = 15 \\ 4y = 9 \\ y = \frac{9}{4} \end{cases}$

2. $\begin{cases} 3x - 2(a - b) = 12 \\ a + 12b = 12 \end{cases}$ $\begin{cases} 3(10) - 2(-10) = 12 \\ 30 + 20 = 12 \\ 50 = 12 \end{cases}$ $\begin{cases} 3a - 2a + 2b = 12 \\ a + 2b = 12 \\ a = 12 - 2b \end{cases}$ $\begin{cases} 3(12) - 2(-10) = 12 \\ 36 + 20 = 12 \\ 56 = 12 \end{cases}$ $\begin{cases} 3a - 2a + 2b = 12 \\ a + 2b = 12 \\ a = 12 - 2b \end{cases}$ $\begin{cases} 3(12) - 2(-10) = 12 \\ 36 + 20 = 12 \\ 56 = 12 \end{cases}$

3. $\begin{cases} 3x + y = 12 \\ x - y = 10 \end{cases}$ $\begin{cases} 3(3) + 4 = 12 \\ 9 + 4 = 12 \\ 13 = 12 \end{cases}$ $\begin{cases} 3x + 2x - 10 = 12 \\ 5x - 10 = 12 \\ 5x = 22 \\ x = \frac{22}{5} \end{cases}$ $\begin{cases} 2(\frac{22}{5}) + 4y = 12 \\ \frac{44}{5} + 4y = 12 \\ 4y = 12 - \frac{44}{5} \\ 4y = \frac{60 - 44}{5} \\ 4y = \frac{16}{5} \\ y = \frac{4}{5} \end{cases}$ $\begin{cases} 3x + 2x - 10 = 12 \\ 5x - 10 = 12 \\ 5x = 22 \\ x = \frac{22}{5} \end{cases}$ $\begin{cases} 2(\frac{22}{5}) + 4y = 12 \\ \frac{44}{5} + 4y = 12 \\ 4y = 12 - \frac{44}{5} \\ 4y = \frac{60 - 44}{5} \\ 4y = \frac{16}{5} \\ y = \frac{4}{5} \end{cases}$

4. $\begin{cases} 2x + 3y - 7 = 4 \\ 3x + 2z - 14 = 4 \\ 3z + 15 = 4 \\ x = 6 \end{cases}$ $\begin{cases} 2(6) + 3y - 7 = 4 \\ 12 + 3y - 7 = 4 \\ 5 + 3y = 4 \\ 3y = -1 \\ y = -\frac{1}{3} \end{cases}$ $\begin{cases} 3x + 2z - 14 = 4 \\ 3(6) + 2z - 14 = 4 \\ 18 + 2z - 14 = 4 \\ 4 + 2z = 4 \\ 2z = 0 \\ z = 0 \end{cases}$ $\begin{cases} 3z + 15 = 4 \\ 3(0) + 15 = 4 \\ 15 = 4 \end{cases}$ $\begin{cases} 2x + 3y - 7 = 4 \\ 3x + 2z - 14 = 4 \\ 3z + 15 = 4 \\ x = 6 \end{cases}$

5. $\begin{cases} 4x = 300 - 2y \\ 6y = 300 \\ y = 50 \\ 2(50) = x \\ x = 100 \end{cases}$ $\begin{cases} 4(9) = 300 - 100 \\ 360 = 200 \\ 200 = 200 \end{cases}$ $\begin{cases} 6(0.5x + 0.25y) = 200 \\ 3x + 1.5y = 200 \\ 3(7) + 1.5(6) = 200 \\ 21 + 9 = 200 \\ 30 = 200 \end{cases}$ $\begin{cases} 50(1-y) + 75y = 200 \\ 50 - 50y + 75y = 200 \\ 50 + 25y = 200 \\ 25y = 150 \\ y = 6 \end{cases}$ $\begin{cases} 50 - 50y + 75y = 200 \\ 50 + 25y = 200 \\ 25y = 150 \\ y = 6 \end{cases}$ $\begin{cases} 50 + 25y = 200 \\ 25y = 150 \\ y = 6 \end{cases}$ $\begin{cases} 50 + 25y = 200 \\ 25y = 150 \\ y = 6 \end{cases}$

7. A quilt maker sews both large and small quilts. A large quilt requires 8 yards of fabric while a small quilt requires 3 yards. How many of each size quilt did she make if she used a total of 90 yards of fabric to make 15 quilts?

$\begin{cases} L = \text{large} \\ S = \text{small} \end{cases}$ quilts

$\begin{cases} 8L + 3S = 90 \\ \frac{8}{8} + \Delta = 15 - \frac{3}{8} \\ \Delta = -\frac{1}{8}(3 + 15) \end{cases}$ $\begin{cases} 8L + 3S = 90 \\ 8L + 3L + 45 = 90 \\ 5L = 45 \\ L = 9 \end{cases}$ $\begin{cases} 8L + 3S = 90 \\ 8(9) + 3S = 90 \\ 72 + 3S = 90 \\ 3S = 18 \\ S = 6 \end{cases}$ $\begin{cases} 8L + 3S = 90 \\ 8(9) + 3(6) = 90 \\ 72 + 18 = 90 \\ 90 = 90 \end{cases}$

6 small & 9 large quilts are made w/ 90 yds of fabric