

**Chapter**  
**7****Maintaining Mathematical Proficiency****Simplify the expression.**

1.  $5x - 6 + 3x$

2.  $3t + 7 - 3t - 4$

3.  $8s - 4 + 4s - 6 - 5s$

4.  $9m + 3 + m - 3 + 5m$

5.  $-4 - 3p - 7 - 3p - 4$

6.  $12(z - 1) + 4$

7.  $-6(x + 2) - 4$

8.  $3(h + 4) - 3(h - 4)$

9.  $7(z + 4) - 3(z + 2) - 2(z - 3)$

**Find the greatest common factor.**

10. 24, 32

11. 30, 55

12. 48, 84

13. 28, 72

14. 42, 60

15. 35, 99

16. Explain how to find the greatest common factor of 42, 70, and 84.

# 7.1

## Adding and Subtracting Polynomials

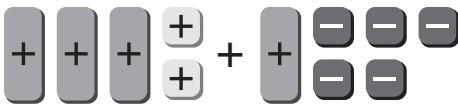
For use with Exploration 7.1


**Essential Question** How can you add and subtract polynomials?

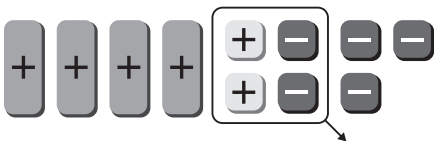
### 1 EXPLORATION: Adding Polynomials

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Write the expression modeled by the algebra tiles in each step.

Step 1   $(3x + 2) + (x - 5)$

Step 2  \_\_\_\_\_

Step 3  \_\_\_\_\_

Step 4  \_\_\_\_\_

### 2 EXPLORATION: Subtracting Polynomials

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Write the expression modeled by the algebra tiles in each step.

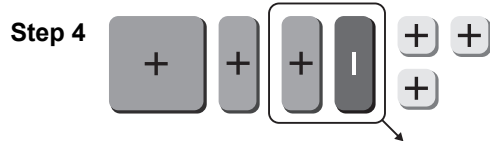
Step 1   $(x^2 + 2x + 2) - (x - 1)$

Step 2  \_\_\_\_\_

Step 3  \_\_\_\_\_

**7.1 Adding and Subtracting Polynomials (continued)**

**2 EXPLORATION: Subtracting Polynomials (continued)**



\_\_\_\_\_



\_\_\_\_\_

**Communicate Your Answer**

3. How can you add and subtract polynomials?

4. Use your methods in Question 3 to find each sum or difference.

a.  $(x^2 + 2x - 1) + (2x^2 - 2x + 1)$

b.  $(4x + 3) + (x - 2)$

c.  $(x^2 + 2) - (3x^2 + 2x + 5)$

d.  $(2x - 3x) - (x^2 - 2x + 4)$

Name \_\_\_\_\_ Date \_\_\_\_\_

## 7.1

### Notetaking with Vocabulary

For use after Lesson 7.1

In your own words, write the meaning of each vocabulary term.

monomial

degree of a monomial

polynomial

binomial

trinomial

degree of a polynomial

standard form

leading coefficient

closed

**Notes:**

**7.1** Notetaking with Vocabulary (continued)**Core Concepts****Polynomials**

A **polynomial** is a monomial or a sum of monomials. Each monomial is called a *term* of the polynomial. A polynomial with two terms is a **binomial**. A polynomial with three terms is a **trinomial**.

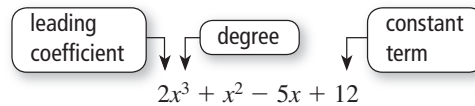
Binomial

$5x + 2$

Trinomial

$x^2 + 5x + 2$

The **degree of a polynomial** is the greatest degree of its terms. A polynomial in one variable is in **standard form** when the exponents of the terms decrease from left to right. When you write a polynomial in standard form, the coefficient of the first term is the **leading coefficient**.

**Notes:****Extra Practice**

In Exercises 1–8, find the degree of the monomial.

1.  $-6s$

2.  $w$

3.  $8$

4.  $-2abc$

5.  $7x^2y$

6.  $4r^2s^3t$

7.  $10mn^3$

8.  $\frac{2}{3}$

**7.1** Notetaking with Vocabulary (continued)

In Exercises 9–12, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.

9.  $x + 3x^2 + 5$

10.  $\sqrt{5}y$

11.  $3x^5 + 6x^8$

12.  $f^2 - 2f + f^4$

In Exercises 13–16, find the sum.

13.  $(-4x + 9) + (6x - 14)$

14.  $(-3a - 2) + (7a + 5)$

15.  $(x^2 + 3x + 5) + (-x^2 + 6x - 4)$

16.  $(t^2 + 3t^3 - 3) + (2t^2 + 7t - 2t^3)$

In Exercises 17–20, find the difference.

17.  $(g - 4) - (3g - 6)$

18.  $(-5h - 2) - (7h + 6)$

19.  $(-x^2 - 5) - (-3x^2 - x - 8)$

20.  $(k^2 + 6k^3 - 4) - (5k^3 + 7k - 3k^2)$

**7.2****Multiplying Polynomials**

For use with Exploration 7.2

**Essential Question** How can you multiply two polynomials?**1 EXPLORATION:** Multiplying Monomials Using Algebra Tiles

Work with a partner. Write each product. Explain your reasoning.

a.  $\boxed{+} \cdot \boxed{+} = \underline{\hspace{2cm}}$

b.  $\boxed{+} \cdot \boxed{-} = \underline{\hspace{2cm}}$

c.  $\boxed{-} \cdot \boxed{-} = \underline{\hspace{2cm}}$

d.  $\boxed{+} \cdot \boxed{+} = \underline{\hspace{2cm}}$

e.  $\boxed{+} \cdot \boxed{-} = \underline{\hspace{2cm}}$

f.  $\boxed{-} \cdot \boxed{+} = \underline{\hspace{2cm}}$

g.  $\boxed{-} \cdot \boxed{-} = \underline{\hspace{2cm}}$

h.  $\boxed{+} \cdot \boxed{+} = \underline{\hspace{2cm}}$

i.  $\boxed{+} \cdot \boxed{-} = \underline{\hspace{2cm}}$

j.  $\boxed{-} \cdot \boxed{-} = \underline{\hspace{2cm}}$

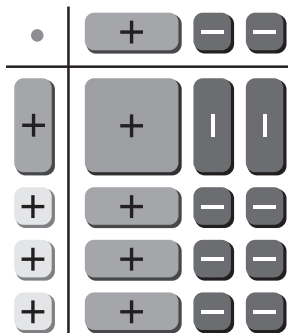
**7.2 Multiplying Polynomials (continued)**

**2 EXPLORATION: Multiplying Binomials Using Algebra Tiles**

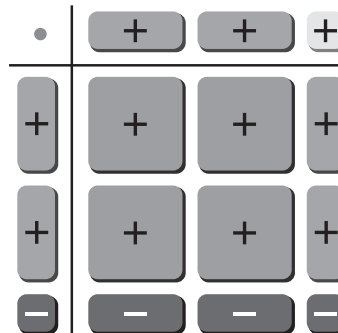
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Write the product of two binomials modeled by each rectangular array of algebra tiles. In parts (c) and (d), first draw the rectangular array of algebra tiles that models each product.

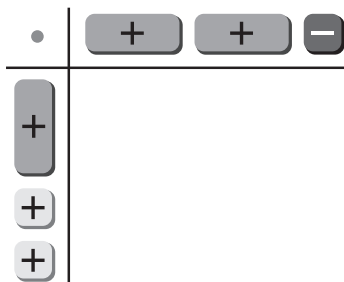
a.  $(x + 3)(x - 2) =$  \_\_\_\_\_



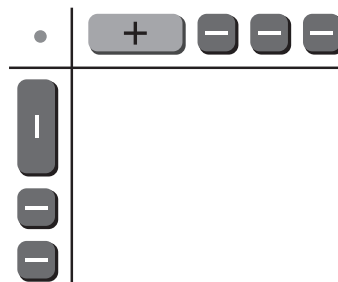
b.  $(2x - 1)(2x + 1) =$  \_\_\_\_\_



c.  $(x + 2)(2x - 1) =$  \_\_\_\_\_



d.  $(-x - 2)(x - 3) =$  \_\_\_\_\_



**Communicate Your Answer**

- How can you multiply two polynomials?
- Give another example of multiplying two binomials using algebra tiles that is similar to those in Exploration 2.



**7.2****Notetaking with Vocabulary**

For use after Lesson 7.2

In your own words, write the meaning of each vocabulary term.

FOIL Method

**Core Concepts****FOIL Method**

To multiply two binomials using the FOIL Method, find the sum of the products of the

First terms,  $\widehat{(x + 1)(x + 2)}$   $\longrightarrow$   $x(x) = x^2$

Outer terms,  $\widehat{(x + 1)(x + 2)}$   $\longrightarrow$   $x(2) = 2x$

Inner terms, and  $\widehat{(x + 1)(x + 2)}$   $\longrightarrow$   $1(x) = x$

Last terms.  $\widehat{(x + 1)(x + 2)}$   $\longrightarrow$   $1(2) = 2$

$$(x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$$

**Notes:**

**7.2** Notetaking with Vocabulary (continued)

**Extra Practice**

In Exercises 1–6, use the Distributive Property to find the product.

1.  $(x - 2)(x - 1)$

2.  $(b - 3)(b + 2)$

3.  $(g + 2)(g + 4)$

4.  $(a - 1)(2a + 5)$

5.  $(3n - 4)(n + 1)$

6.  $(r + 3)(3r + 2)$

In Exercises 7–12, use a table to find the product.

7.  $(x - 3)(x - 2)$


8.  $(y + 1)(y - 6)$


9.  $(q + 3)(q + 7)$


10.  $(2w - 5)(w - 3)$


11.  $(6h - 2)(-3 - 2h)$


12.  $(-3 + 4j)(3j + 4)$


**7.2** Notetaking with Vocabulary (continued)

In Exercises 13–18, use the FOIL Method to find the product.

13.  $(x + 2)(x - 3)$

14.  $(z + 3)(z + 2)$

15.  $(h - 2)(h + 4)$

16.  $(2m - 1)(m + 2)$

17.  $(4n - 1)(3n + 4)$

18.  $(-q - 1)(q + 1)$

In Exercises 19–24, find the product.

19.  $(x - 2)(x^2 + x - 1)$

20.  $(2 - a)(3a^2 + 3a - 5)$

21.  $(h + 1)(h^2 - h - 1)$

22.  $(d + 3)(d^2 - 4d + 1)$

23.  $(3n^2 + 2n - 5)(2n + 1)$

24.  $(2p^2 + p - 3)(3p - 1)$

# 7.3

## Special Products of Polynomials

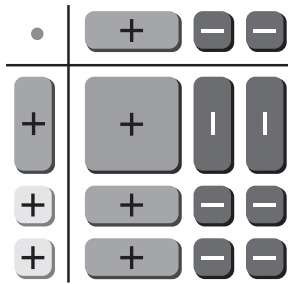
For use with Exploration 7.3

**Essential Question** What are the patterns in the special products  $(a + b)(a - b)$ ,  $(a + b)^2$ , and  $(a - b)^2$ ?

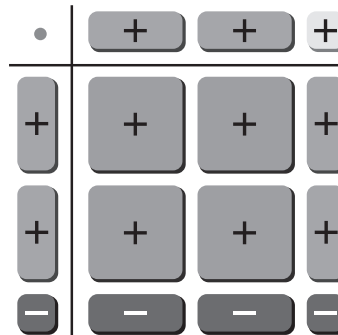
### 1 EXPLORATION: Finding a Sum and Difference Pattern

**Work with a partner.** Write the product of two binomials modeled by each rectangular array of algebra tiles.

a.  $(x + 2)(x - 2) =$  \_\_\_\_\_



b.  $(2x - 1)(2x + 1) =$  \_\_\_\_\_

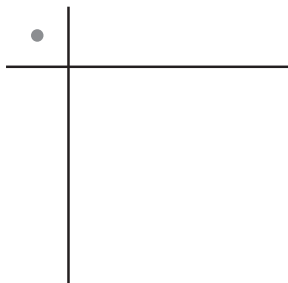


### 2 EXPLORATION: Finding the Square of a Binomial Pattern

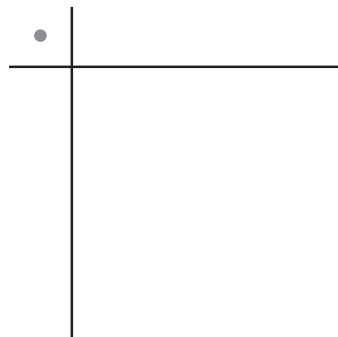
Go to [BigIdeasMath.com](http://BigIdeasMath.com) for an interactive tool to investigate this exploration.

**Work with a partner.** Draw the rectangular array of algebra tiles that models each product of two binomials. Write the product.

a.  $(x + 2)^2 =$  \_\_\_\_\_



b.  $(2x - 1)^2 =$  \_\_\_\_\_



**7.3 Special Products of Polynomials (continued)****Communicate Your Answer**

3. What are the patterns in the special products  $(a + b)(a - b)$ ,  $(a + b)^2$ , and  $(a - b)^2$ ?
4. Use the appropriate special product pattern to find each product. Check your answers using algebra tiles.
- a.  $(x + 3)(x - 3)$       b.  $(x - 4)(x + 4)$       c.  $(3x + 1)(3x - 1)$
- d.  $(x + 3)^2$       e.  $(x - 2)^2$       f.  $(3x + 1)^2$

**7.3****Notetaking with Vocabulary**

For use after Lesson 7.3

In your own words, write the meaning of each vocabulary term.

binomial

**Core Concepts****Square of a Binomial Pattern****Algebra**

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

**Example**

$$\begin{aligned}(x + 5)^2 &= (x)^2 + 2(x)(5) + (5)^2 \\ &= x^2 + 10x + 25\end{aligned}$$

$$\begin{aligned}(2x - 3)^2 &= (2x)^2 - 2(2x)(3) + (3)^2 \\ &= 4x^2 - 12x + 9\end{aligned}$$

**Notes:****Sum and Difference Pattern****Algebra**

$$(a + b)(a - b) = a^2 - b^2$$

**Example**

$$(x + 3)(x - 3) = x^2 - 9$$

**Notes:**

**7.3** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–18, find the product.

1.  $(a + 3)^2$

2.  $(b - 2)^2$

3.  $(c + 4)^2$

4.  $(-2x + 1)^2$

5.  $(3x - 2)^2$

6.  $(-4p - 3)^2$

7.  $(3x + 2y)^2$

8.  $(2a - 3b)^2$

9.  $(-4c + 5d)^2$

10.  $(x - 3)(x + 3)$

11.  $(q + 5)(q - 5)$

12.  $(t - 11)(t + 11)$

**7.3** Notetaking with Vocabulary (continued)

13.  $(5a - 1)(5a + 1)$

14.  $\left(\frac{1}{4}b + 1\right)\left(\frac{1}{4}b - 1\right)$

15.  $\left(\frac{1}{2}c + \frac{1}{3}\right)\left(\frac{1}{2}c - \frac{1}{3}\right)$

16.  $(-m + 2n)(-m - 2n)$

17.  $(-3j - 2k)(-3j + 2k)$

18.  $\left(6a + \frac{1}{2}b\right)\left(-6a + \frac{1}{2}b\right)$

In Exercises 19–24, use special product patterns to find the product.

19.  $18 \cdot 22$

20.  $49 \cdot 51$

21.  $19\frac{3}{5} \cdot 20\frac{2}{5}$

22.  $(31)^2$

23.  $(20.7)^2$

24.  $(109)^2$

25. Find  $k$  so that  $kx^2 - 12x + 9$  is the square of a binomial.



# 7.4

## Solving Polynomial Equations in Factored Form

For use with Exploration 7.4

**Essential Question** How can you solve a polynomial equation?

### 1 EXPLORATION: Matching Equivalent Forms of an Equation

**Work with a partner.** An equation is considered to be in *factored form* when the product of the factors is equal to 0. Match each factored form of the equation with its equivalent standard form and nonstandard form.

**Factored Form**

a.  $(x - 1)(x - 3) = 0$

b.  $(x - 2)(x - 3) = 0$

c.  $(x + 1)(x - 2) = 0$

d.  $(x - 1)(x + 2) = 0$

e.  $(x + 1)(x - 3) = 0$

**Standard Form**

A.  $x^2 - x - 2 = 0$

B.  $x^2 + x - 2 = 0$

C.  $x^2 - 4x + 3 = 0$

D.  $x^2 - 5x + 6 = 0$

E.  $x^2 - 2x - 3 = 0$

**Nonstandard Form**

1.  $x^2 - 5x = -6$

2.  $(x - 1)^2 = 4$

3.  $x^2 - x = 2$

4.  $x(x + 1) = 2$

5.  $x^2 - 4x = -3$

### 2 EXPLORATION: Writing a Conjecture

Go to [BigIdeasMath.com](http://BigIdeasMath.com) for an interactive tool to investigate this exploration.

**Work with a partner.** Substitute 1, 2, 3, 4, 5, and 6 for  $x$  in each equation and determine whether the equation is true. Organize your results in the table. Write a conjecture describing what you discovered.

	Equation	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$
a.	$(x - 1)(x - 2) = 0$						
b.	$(x - 2)(x - 3) = 0$						
c.	$(x - 3)(x - 4) = 0$						
d.	$(x - 4)(x - 5) = 0$						
e.	$(x - 5)(x - 6) = 0$						
f.	$(x - 6)(x - 1) = 0$						

**7.4 Solving Polynomial Equations in Factored Form (continued)****3 EXPLORATION: Special Properties of 0 and 1**

**Work with a partner.** The numbers 0 and 1 have special properties that are shared by no other numbers. For each of the following, decide whether the property is true for 0, 1, both, or neither. Explain your reasoning.

- a. When you add \_\_\_\_ to a number  $n$ , you get  $n$ .
- b. If the product of two numbers is \_\_\_\_, then at least one of the numbers is 0.
- c. The square of \_\_\_\_ is equal to itself.
- d. When you multiply a number  $n$  by \_\_\_\_, you get  $n$ .
- e. When you multiply a number by  $n$  by \_\_\_\_, you get 0.
- f. The opposite of \_\_\_\_ is equal to itself.

**Communicate Your Answer**

4. How can you solve a polynomial equation?
5. One of the properties in Exploration 3 is called the Zero-Product Property. It is one of the most important properties in all of algebra. Which property is it? Why do you think it is called the Zero-Product Property? Explain how it is used in algebra and why it so important.

Name \_\_\_\_\_

Date \_\_\_\_\_

## 7.4

### Notetaking with Vocabulary

For use after Lesson 7.4

In your own words, write the meaning of each vocabulary term.

factored form

Zero-Product Property

roots

repeated roots

### Core Concepts

#### Zero-Product Property

**Words** If the product of two real numbers is 0, then at least one of the numbers is 0.

**Algebra** If  $a$  and  $b$  are real numbers and  $ab = 0$ , then  $a = 0$  or  $b = 0$ .

**Notes:**

**7.4** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–12, solve the equation.

1.  $x(x + 5) = 0$

2.  $a(a - 12) = 0$

3.  $5p(p - 2) = 0$

4.  $(c - 2)(c + 1) = 0$

5.  $(2b - 6)(3b + 18) = 0$

6.  $(3 - 5s)(-3 + 5s) = 0$

7.  $(x - 3)^2 = 0$

8.  $(3d + 7)(5d - 6) = 0$

9.  $(2t + 8)(2t - 8) = 0$

10.  $(w + 4)^2(w + 1) = 0$

11.  $g(6 - 3g)(6 + 3g) = 0$

12.  $(4 - m)\left(8 + \frac{2}{3}m\right)(-2 - 3m) = 0$

**7.4 Notetaking with Vocabulary (continued)**

In Exercises 13–18, factor the polynomial.

13.  $6x^2 + 3x$

14.  $4y^4 - 20y^3$

15.  $18u^4 - 6u$

16.  $7z^7 + 2z^6$

17.  $24h^3 + 8h$

18.  $15f^4 - 45f$

In Exercises 19–24, solve the equation.

19.  $6k^2 + k = 0$

20.  $35n - 49n^2 = 0$

21.  $4z^2 + 52z = 0$

22.  $6x^2 = -72x$

23.  $22s = 11s^2$

24.  $7p^2 = 21p$

25. A boy kicks a ball in the air. The height  $y$  (in feet) above the ground of the ball is modeled by the equation  $y = -16x^2 + 80x$ , where  $x$  is the time (in seconds) since the ball was kicked. Find the roots of the equation when  $y = 0$ . Explain what the roots mean in this situation.

# 7.5

## Factoring $x^2 + bx + c$

For use with Exploration 7.5

**Essential Question** How can you use algebra tiles to factor the trinomial  $x^2 + bx + c$  into the product of two binomials?

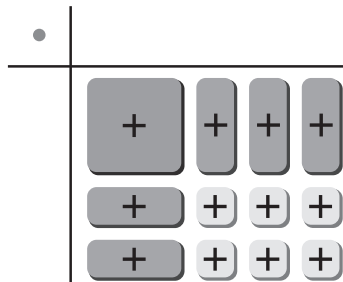
### 1 EXPLORATION: Finding Binomial Factors

Go to [BigIdeasMath.com](http://BigIdeasMath.com) for an interactive tool to investigate this exploration.

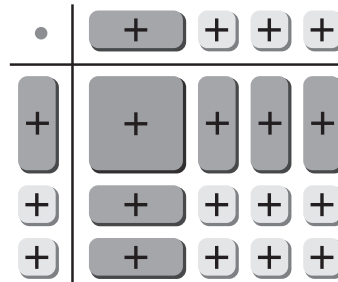
**Work with a partner.** Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

**Sample**  $x^2 + 5x + 6$

**Step 1** Arrange algebra tiles that model  $x^2 + 5x + 6$  into a rectangular array.



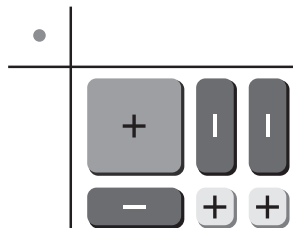
**Step 2** Use additional algebra tiles to model the dimensions of the rectangle.



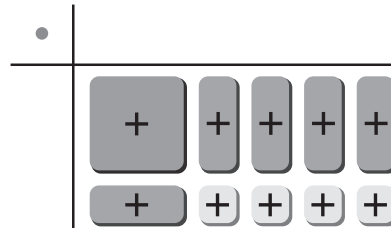
**Step 3** Write the polynomial in factored form using the dimensions of the rectangle.

width      length  
 $\text{Area} = x^2 + 5x + 6 = (x + 2)(x + 3)$

a.  $x^2 - 3x + 2 =$  \_\_\_\_\_



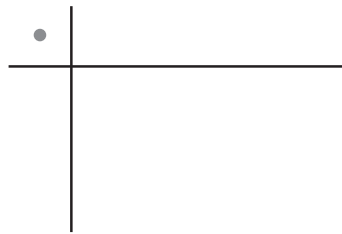
b.  $x^2 + 5x + 4 =$  \_\_\_\_\_



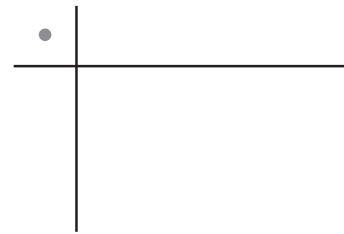
**7.5** Factoring  $x^2 + bx + c$  (continued)

**1** **EXPLORATION:** Finding Binomial Factors (continued)

c.  $x^2 - 7x + 12 =$  \_\_\_\_\_



d.  $x^2 + 7x + 12 =$  \_\_\_\_\_



**Communicate Your Answer**

2. How can you use algebra tiles to factor the trinomial  $x^2 + bx + c$  into the product of two binomials?
  
3. Describe a strategy for factoring the trinomial  $x^2 + bx + c$  that does not use algebra tiles.

**7.5****Notetaking with Vocabulary**

For use after Lesson 7.5

In your own words, write the meaning of each vocabulary term.

polynomial

FOIL Method

Zero-Product Property

**Core Concepts****Factoring  $x^2 + bx + c$  When  $c$  Is Positive****Algebra**  $x^2 + bx + c = (x + p)(x + q)$  when  $p + q = b$  and  $pq = c$ .When  $c$  is positive,  $p$  and  $q$  have the same sign as  $b$ .**Examples**  $x^2 + 6x + 5 = (x + 1)(x + 5)$ 

$$x^2 - 6x + 5 = (x - 1)(x - 5)$$

**Notes:****Factoring  $x^2 + bx + c$  When  $c$  Is Negative****Algebra**  $x^2 + bx + c = (x + p)(x + q)$  when  $p + q = b$  and  $pq = c$ .When  $c$  is negative,  $p$  and  $q$  have different signs.**Example**  $x^2 - 4x - 5 = (x + 1)(x - 5)$ **Notes:**



**7.5** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–12, factor the polynomial.

1.  $c^2 + 8c + 7$

2.  $a^2 + 16a + 64$

3.  $x^2 + 11x + 18$

4.  $d^2 + 6d + 8$

5.  $s^2 + 11s + 10$

6.  $u^2 + 10u + 9$

7.  $b^2 + 3b - 54$

8.  $y^2 - y - 2$

9.  $u + 3u - 18$

10.  $z^2 - z - 56$

11.  $h^2 + 2h - 24$

12.  $f^2 - 3f - 40$

**7.5** Notetaking with Vocabulary (continued)

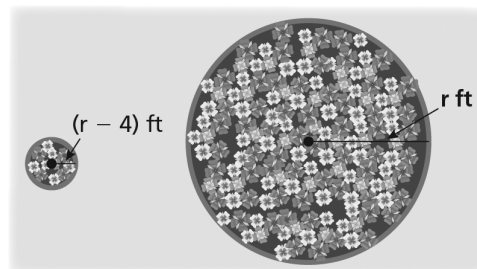
In Exercises 13–18, solve the equation.

13.  $g^2 - 13g + 40 = 0$       14.  $k^2 - 5k + 6 = 0$       15.  $w^2 - 7w + 10 = 0$

16.  $x^2 - x = 30$       17.  $r^2 - 3r = -2$       18.  $t^2 - 7t = 8$

19. The area of a right triangle is 32 square miles. One leg of the triangle is 4 miles longer than the other leg. Find the length of each leg.

20. You have two circular flower beds, as shown. The sum of the areas of the two flower beds is  $136\pi$  square feet. Find the radius of each bed.



**7.6**

**Factoring  $ax^2 + bx + c$**   
For use with Exploration 7.6

**Essential Question** How can you use algebra tiles to factor the trinomial  $ax^2 + bx + c$  into the product of two binomials?

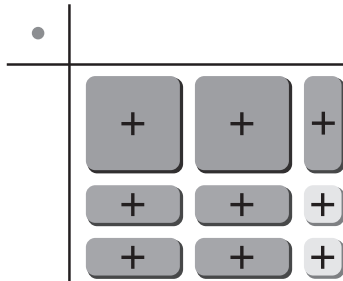
**1 EXPLORATION: Finding Binomial Factors**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

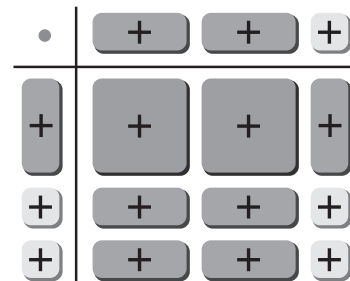
**Work with a partner.** Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

**Sample**  $2x^2 + 5x + 2$

**Step 1** Arrange algebra tiles that model  $2x^2 + 5x + 2$  into a rectangular array.



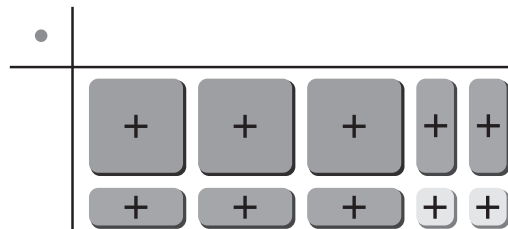
**Step 2** Use additional algebra tiles to model the dimensions of the rectangle.



**Step 3** Write the polynomial in factored form using the dimensions of the rectangle.

width      length  
 $\text{Area} = 2x^2 + 5x + 2 = (x + 2)(2x + 1)$

a.  $3x^2 + 5x + 2 =$  \_\_\_\_\_

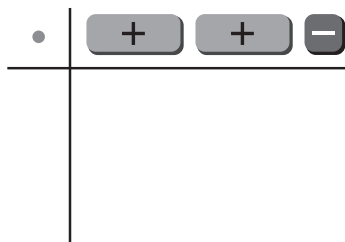


Name \_\_\_\_\_ Date \_\_\_\_\_

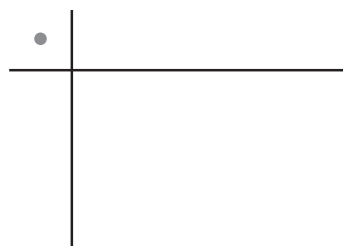
**7.6** Factoring  $ax^2 + bx + c$  (continued)

**1** **EXPLORATION:** Finding Binomial Factors (continued)

b.  $4x^2 + 4x - 3 =$  \_\_\_\_\_



c.  $2x^2 - 11x + 5 =$  \_\_\_\_\_



**Communicate Your Answer**

- How can you use algebra tiles to factor the trinomial  $ax^2 + bx + c$  into the product of two binomials?
  
  
  
  
  
  
  
  
  
  
- Is it possible to factor the trinomial  $2x^2 + 2x + 1$ ? Explain your reasoning.

Name \_\_\_\_\_ Date \_\_\_\_\_

## 7.6

### Notetaking with Vocabulary

For use after Lesson 7.6

In your own words, write the meaning of each vocabulary term.

polynomial

greatest common factor (GCF)

Zero-Product Property

**Notes:**

Name \_\_\_\_\_ Date \_\_\_\_\_

**7.6** Notetaking with Vocabulary (continued)

**Extra Practice**

In Exercises 1–18, factor the polynomial.

1.  $2c^2 - 14c - 36$

2.  $4a^2 + 8a - 140$

3.  $3x^2 - 6x - 24$

4.  $2d^2 - 2d - 60$

5.  $5s^2 + 55s + 50$

6.  $3q^2 + 30q + 27$

7.  $12g^2 - 37g + 28$

8.  $6k^2 - 11k + 4$

9.  $9w^2 + 9w + 2$

10.  $12a^2 + 5a - 2$

11.  $15b^2 + 14b - 8$

12.  $5t^2 + 12t - 9$

Name \_\_\_\_\_ Date \_\_\_\_\_

**7.6** Notetaking with Vocabulary (continued)

13.  $-12b^2 + 5b + 2$

14.  $-6x^2 + x + 15$

15.  $-60g^2 - 11g + 1$

16.  $-2d^2 - d + 6$

17.  $-3r^2 - 4r - 1$

18.  $-8x^2 + 14x - 5$

19. The length of a rectangular shaped park is  $(3x + 5)$  miles. The width is  $(2x + 8)$  miles. The area of the park is 360 square miles. What are the dimensions of the park?

20. The sum of two numbers is 8. The sum of the squares of the two numbers is 34. What are the two numbers?

# 7.7

## Factoring Special Products

For use with Exploration 7.7

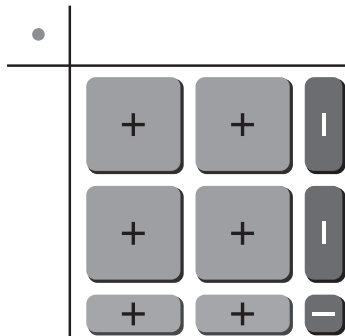
**Essential Question** How can you recognize and factor special products?

### 1 EXPLORATION: Factoring Special Products

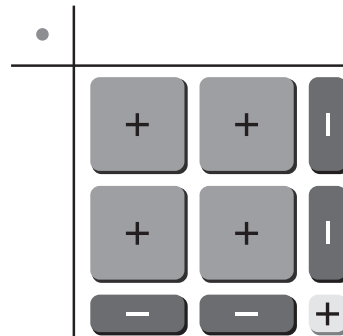
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying. State whether the product is a “special product” that you studied in Section 7.3.

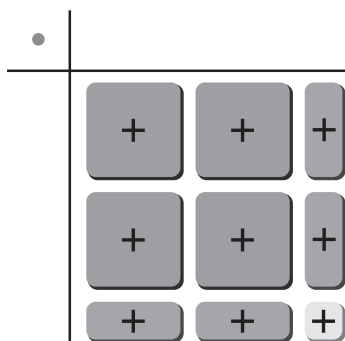
a.  $4x^2 - 1 =$  \_\_\_\_\_



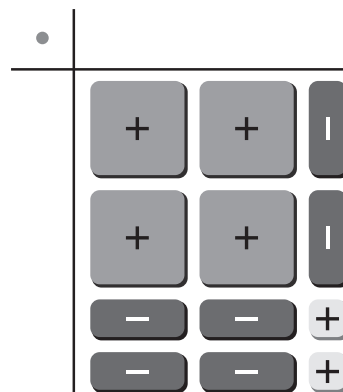
b.  $4x^2 - 4x + 1 =$  \_\_\_\_\_



c.  $4x^2 + 4x + 1 =$  \_\_\_\_\_



d.  $4x^2 - 6x + 2 =$  \_\_\_\_\_



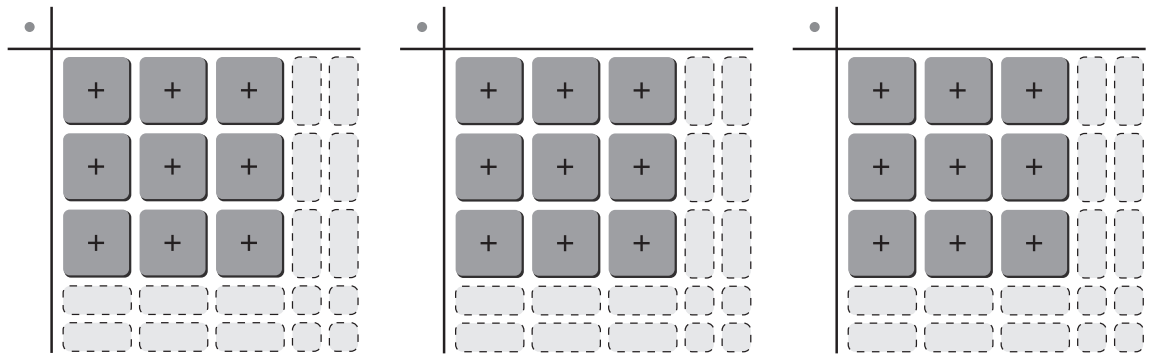


**7.7 Factoring Special Products (continued)**

**2 EXPLORATION: Factoring Special Products**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use algebra tiles to complete the rectangular arrays in three different ways, so that each way represents a different special product. Write each special product in standard form and in factored form.



**Communicate Your Answer**

3. How can you recognize and factor special products? Describe a strategy for recognizing which polynomials can be factored as special products.
  
4. Use the strategy you described in Question 3 to factor each polynomial.
  - a.  $25x^2 + 10x + 1$
  - b.  $25x^2 - 10x + 1$
  - c.  $25x^2 - 1$

**7.7****Notetaking with Vocabulary**

For use after Lesson 7.7

In your own words, write the meaning of each vocabulary term.

polynomial

trinomial

**Core Concepts****Difference of Two Squares Pattern****Algebra**

$$a^2 - b^2 = (a + b)(a - b)$$

**Example**

$$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$$

**Notes:****Perfect Square Trinomial Pattern****Algebra**

$$a^2 + 2ab + b^2 = (a + b)^2$$

**Example**

$$\begin{aligned}x^2 + 6x + 9 &= x^2 + 2(x)(3) + 3^2 \\ &= (x + 3)^2\end{aligned}$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$\begin{aligned}x^2 - 6x + 9 &= x^2 - 2(x)(3) + 3^2 \\ &= (x - 3)^2\end{aligned}$$

**Notes:**

**7.7** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–6, factor the polynomial.

1.  $s^2 - 49$

2.  $t^2 - 81$

3.  $16 - x^2$

4.  $4g^2 - 25$

5.  $36h^2 - 121$

6.  $81 - 49k^2$

In Exercises 7–12, use a special product pattern to evaluate the expression.

7.  $57^2 - 53^2$

8.  $38^2 - 32^2$

9.  $68^2 - 64^2$

10.  $45^2 - 40^2$

11.  $79^2 - 71^2$

12.  $86^2 - 84^2$

**7.7** Notetaking with Vocabulary (continued)

In Exercises 13–18, factor the polynomial.

13.  $x^2 + 16x + 64$

14.  $p^2 + 28p + 196$

15.  $r^2 - 26r + 169$

16.  $a^2 - 18a + 81$

17.  $36c^2 + 84c + 49$

18.  $100x^2 - 20x + 1$

In Exercises 19–24, solve the equation.

19.  $x^2 - 144 = 0$

20.  $9y^2 = 49$

21.  $c^2 + 14c + 49 = 0$

22.  $d^2 - 4d + 4 = 0$

23.  $n^2 + \frac{2}{3}n = -\frac{1}{9}$

24.  $-\frac{6}{5}k + \frac{9}{25} = -k^2$

25. The dimensions of a rectangular prism are  $(x + 1)$  feet by  $(x + 2)$  feet by 4 feet. The volume of the prism is  $(24x - 1)$  cubic feet. What is the value of  $x$ ?

**7.8**

**Factoring Polynomials Completely**  
For use with Exploration 7.8

**Essential Question** How can you factor a polynomial completely?

**1 EXPLORATION: Writing a Product of Linear Factors**

**Work with a partner.** Write the product represented by the algebra tiles. Then multiply to write the polynomial in standard form.

a.  $(\text{+} \text{+})(\text{+} \text{+})(\text{-} \text{-})$

b.  $(\text{+} \text{+} \text{+})(\text{+} \text{+})(\text{-})$

c.  $(\text{+} \text{+} \text{+} \text{+})(\text{+})(\text{+} \text{+})$

d.  $(\text{+} \text{+})(\text{+} \text{-})(\text{+})$

e.  $(\text{-} \text{+})(\text{+} \text{+})(\text{-})$

f.  $(\text{-} \text{-})(\text{+} \text{+})(\text{-} \text{-})$

**2 EXPLORATION: Matching Standard and Factored Forms**

**Work with a partner.** Match the standard form of the polynomial with the equivalent factored form on the next page. Explain your strategy.

a.  $x^3 + x^2$

b.  $x^3 - x$

c.  $x^3 + x^2 - 2x$

d.  $x^3 - 4x^2 + 4x$

e.  $x^3 - 2x^2 - 3x$

f.  $x^3 - 2x^2 + x$

g.  $x^3 - 4x$

h.  $x^3 + 2x^2$

i.  $x^3 - x^2$

j.  $x^3 - 3x^2 + 2x$

k.  $x^3 + 2x^2 - 3x$

l.  $x^3 - 4x^2 + 3x$

m.  $x^3 - 2x^2$

n.  $x^3 + 4x^2 + 4x$

o.  $x^3 + 2x^2 + x$

**7.8 Factoring Polynomials Completely (continued)****2 EXPLORATION: Matching Standard and Factored Forms (continued)**

A.  $x(x + 1)(x - 1)$

B.  $x(x - 1)^2$

C.  $x(x + 1)^2$

D.  $x(x + 2)(x - 1)$

E.  $x(x - 1)(x - 2)$

F.  $x(x + 2)(x - 2)$

G.  $x(x - 2)^2$

H.  $x(x + 2)^2$

I.  $x^2(x - 1)$

J.  $x^2(x + 1)$

K.  $x^2(x - 2)$

L.  $x^2(x + 2)$

M.  $x(x + 3)(x - 1)$

N.  $x(x + 1)(x - 3)$

O.  $x(x - 1)(x - 3)$

**Communicate Your Answer**

3. How can you factor a polynomial completely?

4. Use your answer to Question 3 to factor each polynomial completely.

a.  $x^3 + 4x^2 + 3x$

b.  $x^3 - 6x^2 + 9x$

c.  $x^3 + 6x^2 + 9x$

**7.8****Notetaking with Vocabulary**

For use after Lesson 7.8

In your own words, write the meaning of each vocabulary term.

factoring by grouping

factored completely

**Core Concepts****Factoring by Grouping**

To factor a polynomial with four terms, group the terms into pairs. Factor the GCF out of each pair of terms. Look for and factor out the common binomial factor. This process is called **factoring by grouping**.

**Notes:****Guidelines for Factoring Polynomials Completely**

To factor a polynomial completely, you should try each of these steps.

1. Factor out the greatest common monomial factor.  $3x^2 + 6x = 3x(x + 2)$
- 

2. Look for a difference of two squares or a perfect square trinomial.  $x^2 + 4x + 4 = (x + 2)^2$
- 

3. Factor a trinomial of the form  $ax^2 + bx + c$  into a product of binomial factors.  $3x^2 - 5x - 2 = (3x + 1)(x - 2)$
- 

4. Factor a polynomial with four terms by grouping.  $x^3 + x - 4x^2 - 4 = (x^2 + 1)(x - 4)$
- 

**Notes:**

Name \_\_\_\_\_ Date \_\_\_\_\_

**7.8** Notetaking with Vocabulary (continued)

**Extra Practice**

In Exercises 1–8, factor the polynomial by grouping.

1.  $b^3 - 4b^2 + b - 4$

2.  $ac + ad + bc + bd$

3.  $d^2 + 2c + cd + 2d$

4.  $5t^3 + 6t^2 + 5t + 6$

5.  $8s^3 + s - 64s^2 - 8$

6.  $12a^3 + 2a^2 - 30a - 5$

7.  $4x^3 - 12x^2 - 5x + 15$

8.  $21h^3 + 18h^2 - 35h - 30$



**7.8** Notetaking with Vocabulary (continued)

In Exercises 9–16, factor the polynomial completely.

9.  $4c^3 - 4c$

10.  $100x^4 - 25x^2$

11.  $2a^2 + 3a - 2$

12.  $9x^2 + 3x - 14$

13.  $20p^2 + 22p - 12$

14.  $12x^2 - 20x - 48$

15.  $3s^3 + 2s^2 - 21s - 14$

16.  $2t^4 + t^3 - 10t - 5$

In Exercises 17–22, solve the equation.

17.  $3x^2 - 21x + 30 = 0$

18.  $5y^2 - 5y - 30 = 0$

19.  $c^4 - 81c^2 = 0$

20.  $9d + 9 = d^3 + d^2$

21.  $48n - 3n^2 = 0$

22.  $x^3 + 3x^2 = 16x + 48$